4.1 Signal Induction

Consider an infinite metal plane segmented into strips of width a. We want to calculate the charge induced on the strip by a unit charge located in some distance D above the metal plane / strip. For the calculations below, a mathematics tool like Mathematica (available in the CIP pool), Maple,...can be used. In this exercise, we are not interested in absolute values so that you can omit physical constants like ϵ_0.

1. An infinite conducting sheet is located in the $x-y$-plane. A unit charge q is located at $(0,0,D)$. Calculate the surface charge density $\sigma(x,y,D)$. The correct field configurations (with all $E(x,y,0)$ being perpendicular to the plane) can be found by adding the electric fields of q and of a 'mirror charge' $-q$ at $(0,0,-D)$.
2. Show (by integration) that the total charge on the plane equals q.
3. Calculate the charge confined in a strip at the origin along y of width a, i.e. extending within $-a/2 < x < a/2$. Plot the induced charge as a function of D for various strip widths.
4. How much charge is induced on the strip when the distance of the charge from the plane equals the strip width? What if the distance is half the strip width?
5. Now calculate the charge for the more general case of a strip of width a at an *offset* position x_c. Plot the induced charge vs offset position, for instance for $a = 1$ and $D = 1, 2, 5$.
6. Now we let a charge drift from a large z-position D towards the plane at $z = 0$ with constant velocity. Let the charge drift at $x = y = 0$. Plot the signal of a strip centered at the origin and the signal of its neighbor.

4.2 Photon Absorption

A detector uses a n-type substrate with a bulk resistivity of $\rho = 1\,k\Omega cm$. A highly doped p-junction of $T = 0.5\,\mu$m thickness forms the entrance window for photons. We want to detect red light with a wavelength of $800\,nm$ which has an absorption coefficient of $\approx 6.3 \times 10^3\,cm^{-1}$. We neglect reflection at the surface and do not take into account diffusion of charge carriers in undepleted regions.

1. How much light is lost in the dead layer?
2. What bias voltage is needed to collect 90% of the remaining photo electrons?

4.3 Diffusion

We consider a depleted silicon detector with thickness $d = 300\,\mu$m, depletion voltage $V_{\text{dep}} = 100\,V$ and an applied Bias voltage $V > V_{\text{dep}}$. We use $\mu \approx 1400\,cm^2/Vs$ for the electron mobility. In the lecture, the drift time and the width of the diffusion cloud were estimated by using an *average* field $\langle E\rangle = V/d$. We found $t_{av} = \frac{d^2}{\mu V}$ and $\sigma_{av} = d\sqrt{2U_{Th}/V}$. We want to check how well these approximations are valid.

1. Write down an expression for the field $E(x)$ in the depth x. The junction side is at $x = 0$.

2. Assuming non-saturated drift, i.e. $v = \mu E$, calculate the time it takes for an electron released at x_0 to drift to $x = 0$. (Hint: Solve $v(t) = dx(t)/dt$ for $dx(t)$ and integrate from 0 to x_0.)

3. Does the expression yield a plausible result for $V \to \mathrm{V_{dep}}$?

4. Compare with t_{av} for V just above depletion and for a much higher voltage.

5. Using $\sigma = \sqrt{2Dt}$, what is the charge cloud width σ? How does it compare to the naive guess σ_{av}?

6. Optional: Can you find a closed for expression for saturated drift, using $v(E) = \mu\, E_{crit}/(1+E_{crit}/E)$ with $E_{crit} = 0.7V/\,\mu\mathrm{m}$?

4.4 Charge Explosion by Electrostatic Repulsion

We want to estimate weather the electrostatic repulsion of a charge cloud ('charge explosion') has a significant contribution wrt. diffusion. We assume that - after separation of electrons and holes - the charge of N electrons Nq is confined in a sphere of radius $r(t)$.

1. What is the field at the surface of the sphere? This just follows from Gauss' law!

2. This field leads to an outward-going drift of the electrons, i.e. an expansion of the sphere. Can we assume normal, non-saturated drift? Compare for instance the field for $N = 1000$ and $r = 0.1\,\mu\mathrm{m}$ to the critical field of $1V/\,\mu\mathrm{m}$.

3. To get $r(t)$, you can start from $v = dr/dt$ or $dt = dr/v$ (where we totally neglect the spherical geometry!). Integrate from $rstart = 0$ to some $rstop$ and solve the result for $r(t) = rstop(t)$. Plot $r(t)$ for instance for $N = 1000$ for some nanoseconds, which are typical drift times in a PiN diode.