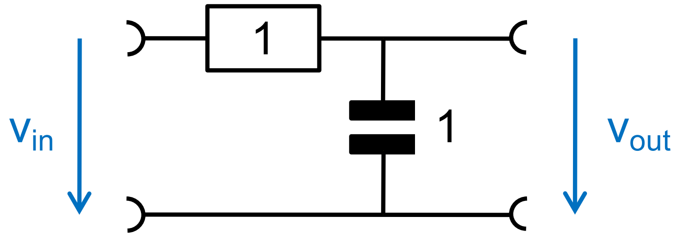
# (Mainly) For Fun:
# Higher Order Filters

(Mathematica file: CCS_HigherOrderFilters.nb)

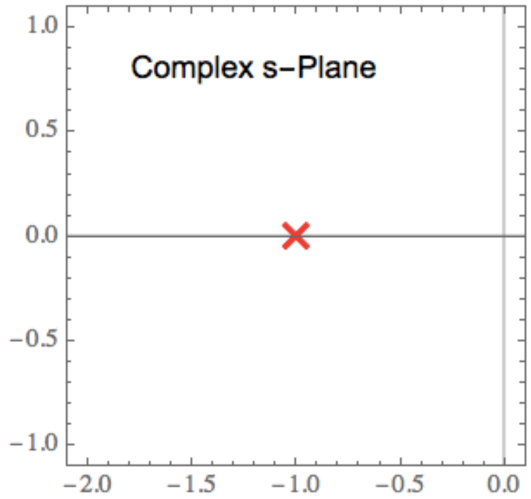
# Reminder: One Low Pass

- (For simplicity, we use fixed values for R and C, often 1 Ω/F)




$$H_{LP1}[s\_] = \frac{1}{1+s}$$

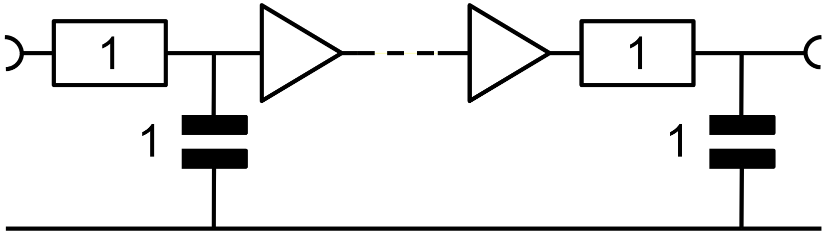
- Mathematically, the function $H_{LP1}[s]$ has a ***POLE*** at s = -1.
- This can be illustrated in the COMPLEX s-Plane:




- This particular pole is ***real***, i.e. it lies on the real axis

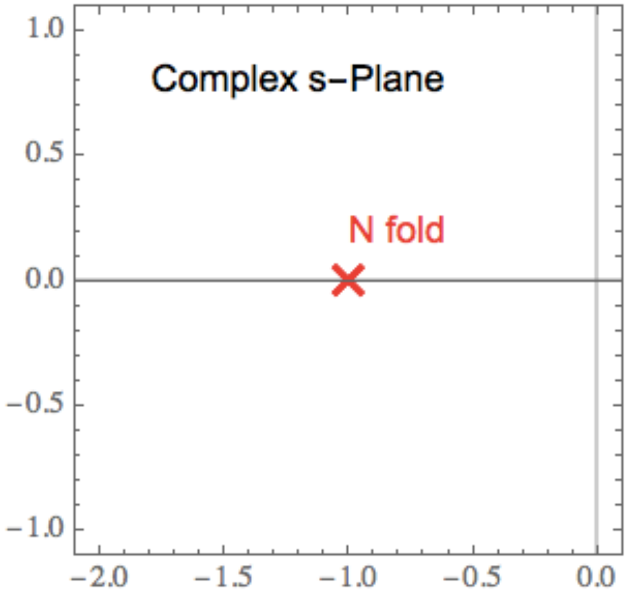
# Reminder: Cascaded Low Pass Stages

- If we cascade N stages *with buffers*, we get



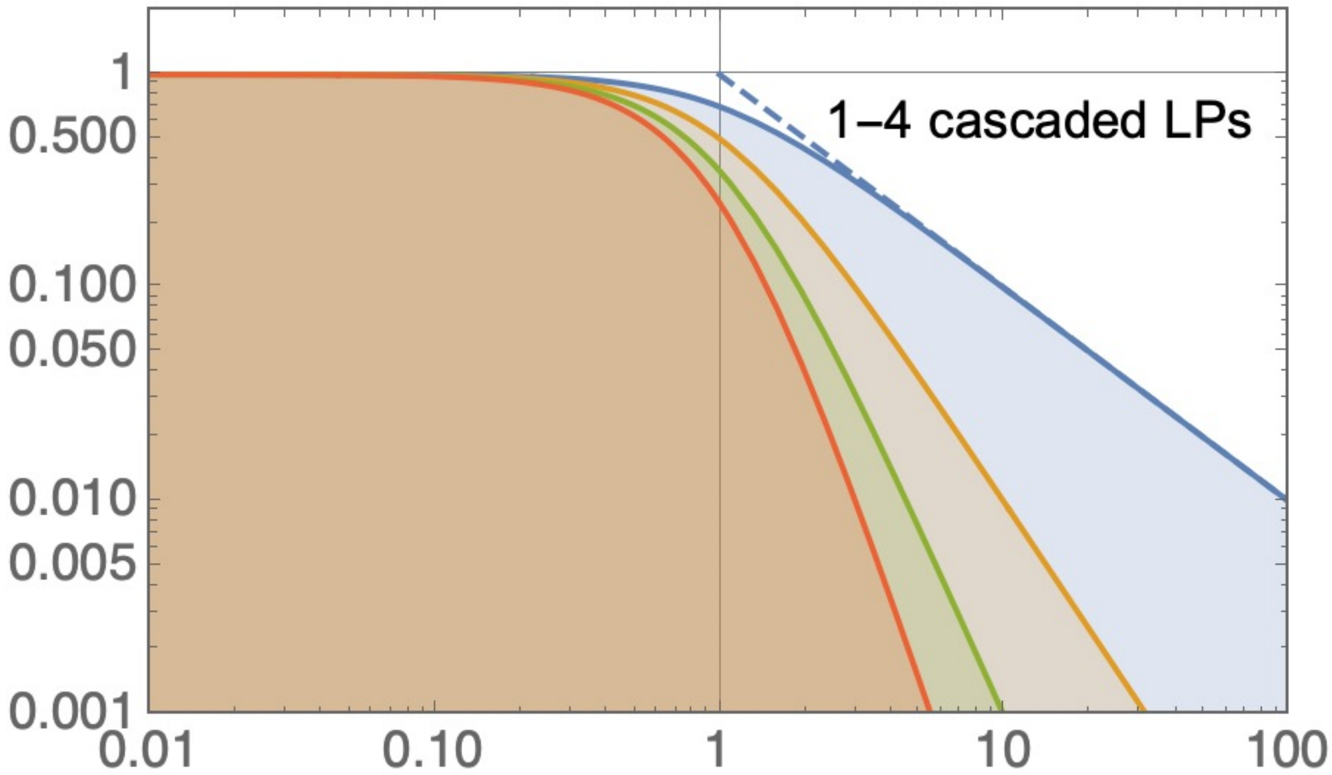

$$H_{LPN}[s\_] = \frac{1}{(1+s)^N}$$

- $H_{LPN}[s]$ has a ***N-fold*** POLE at the same location s = -1.



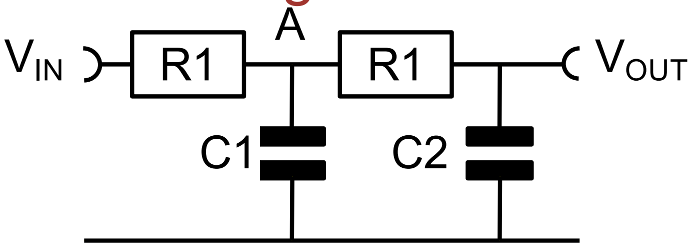
# Bode Plot of Cascaded Low Pass Stages

- The simple Low-Pass rolls off with 6dB/Octave (slope -1)
- Every further Low-Pass adds another 6dB/Octave



# Two Unbuffered Low Pass Stages

- If we cascade two stages *without a buffer*,




the transfer function gets more complicated.

```
EQ1 = (vin - vA)/R1 == (vA - vout)/R2 + vA s C1; (* node A *) EQ2 = (vA - vout)/R2 == vout s C2; (* node Vout *)

Eliminate[{EQ1, EQ2}, vA] // Simplify

vin == (1 + C2 (R1 + R2) s + C1 R1 s (1 + C2 R2 s)) vout

vout/vin /. First@Solve[%, vout]
```

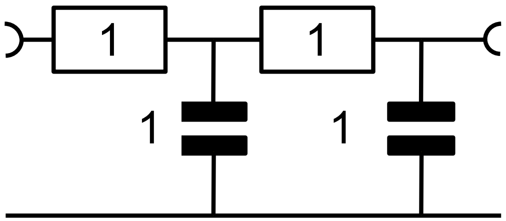
$$\frac{1}{1 + C1\,R1\,s + C2\,R1\,s + C2\,R2\,s + C1\,C2\,R1\,R2\,s^2}$$

```
% /. {R1 → 1, R2 → 1, C1 → 1, C2 → 1} // Highlighted
```

$$\frac{1}{1 + 3\,s + s^2}$$

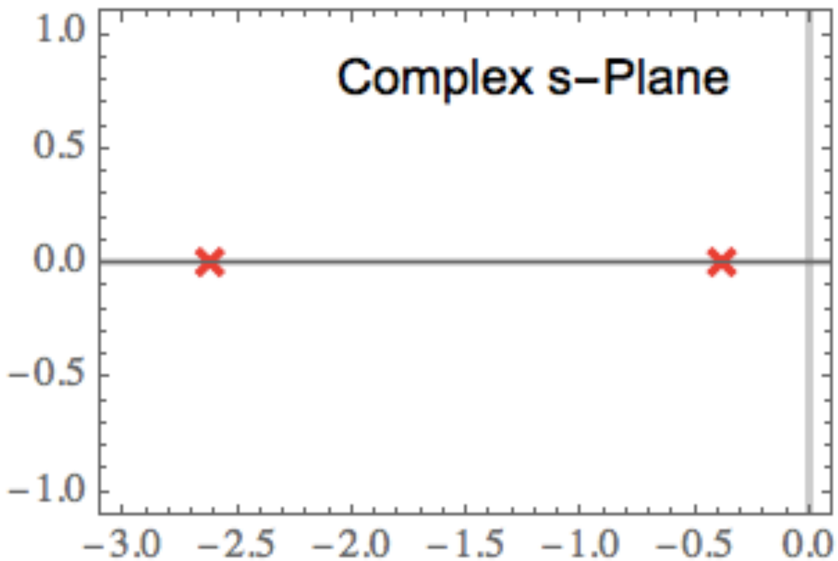
# Two Unbuffered Low Pass Stages

- For R1=R2=1 and C1=C2=1, we get




$$H_{LPCascl}[s] = \frac{1}{1 + 3\,s + s^2}$$

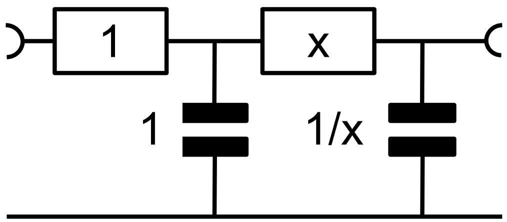
- We now have ***two different*** (still real) poles:




(Their exact locations depend on R/C ratios..)

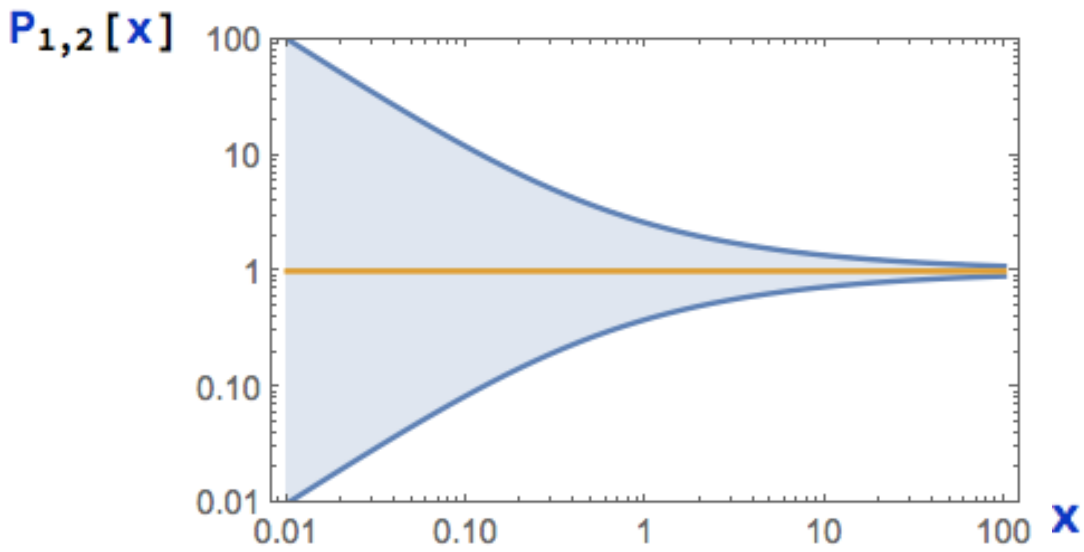
# (Pole Location for Previous Case)

- If we modify R,C of the second stage, keeping RC =1, we get




$$H[s\_] = \frac{x}{s + x + 2\,s\,x + s^2\,x}$$

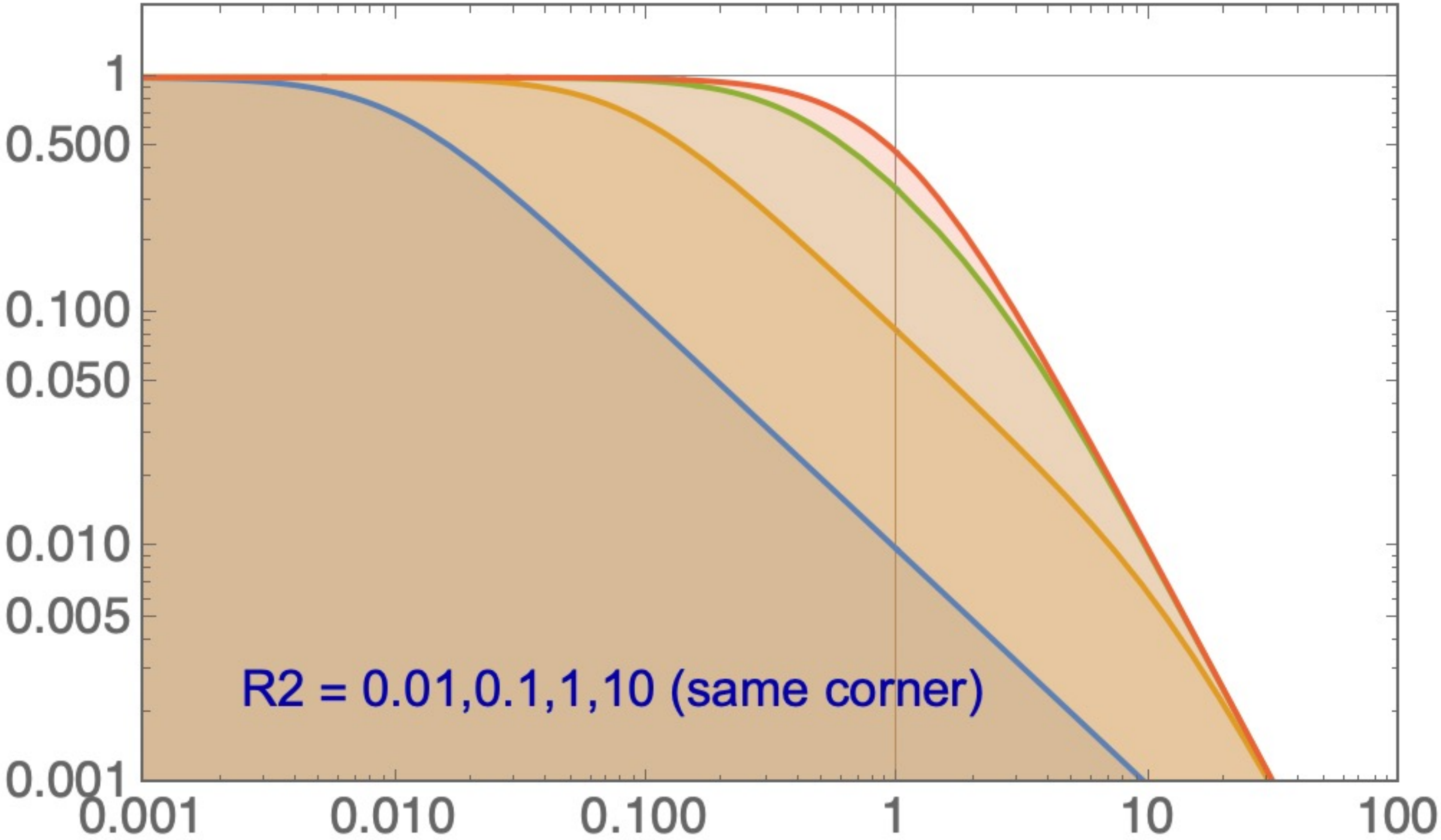
- The poles are at $P_{1,2}[x] = \frac{-1 - 2\,x \pm \sqrt{1 + 4\,x}}{2\,x}$




(when x is large, the 2nd LP does not load the 1st)

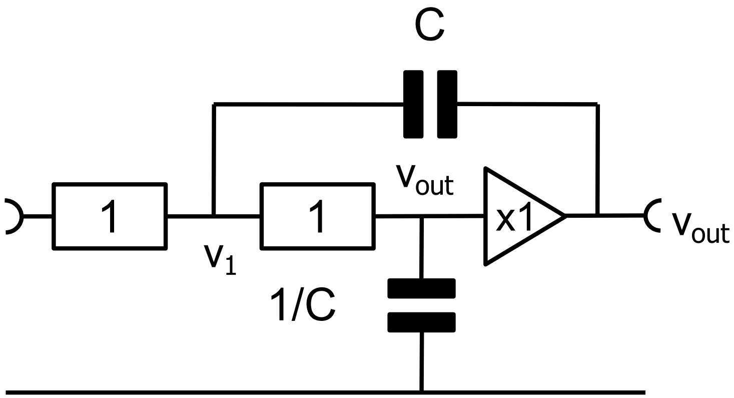
# Bode Plot of 2 Unbuffered Low Pass Stages

- The 2 different real poles lead to kinks at two frequencies



# An Active Filter

- Now consider the following filter ('Sallen and Key')



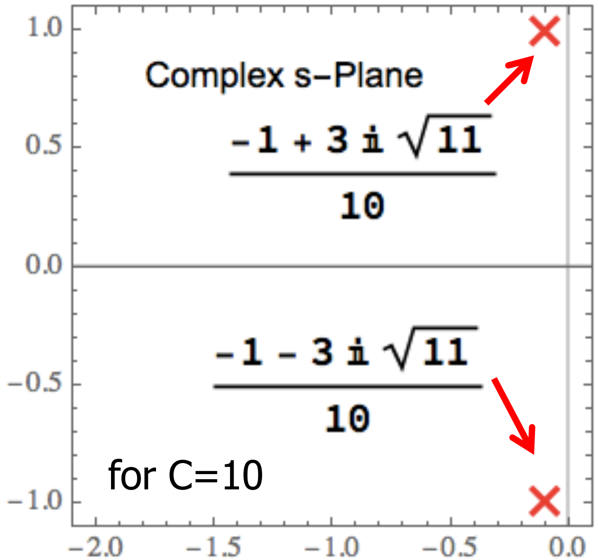

$$EQ1 = \frac{vin - v1}{1} == \frac{v1 - vout}{1} + (v1 - vout)\, s\, 10;$$

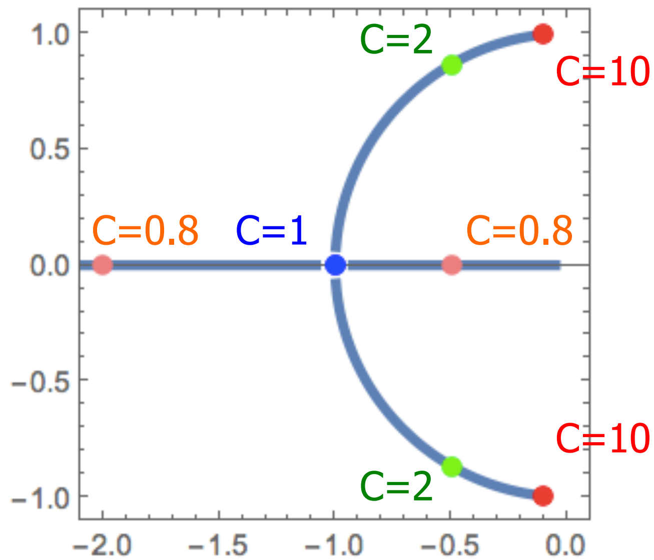
$$EQ2 = \frac{v1 - vout}{1} == vout\, s\, \frac{1}{10};$$

for C=10

$$H[s] = \frac{5}{5 + s + 5 s^2}$$

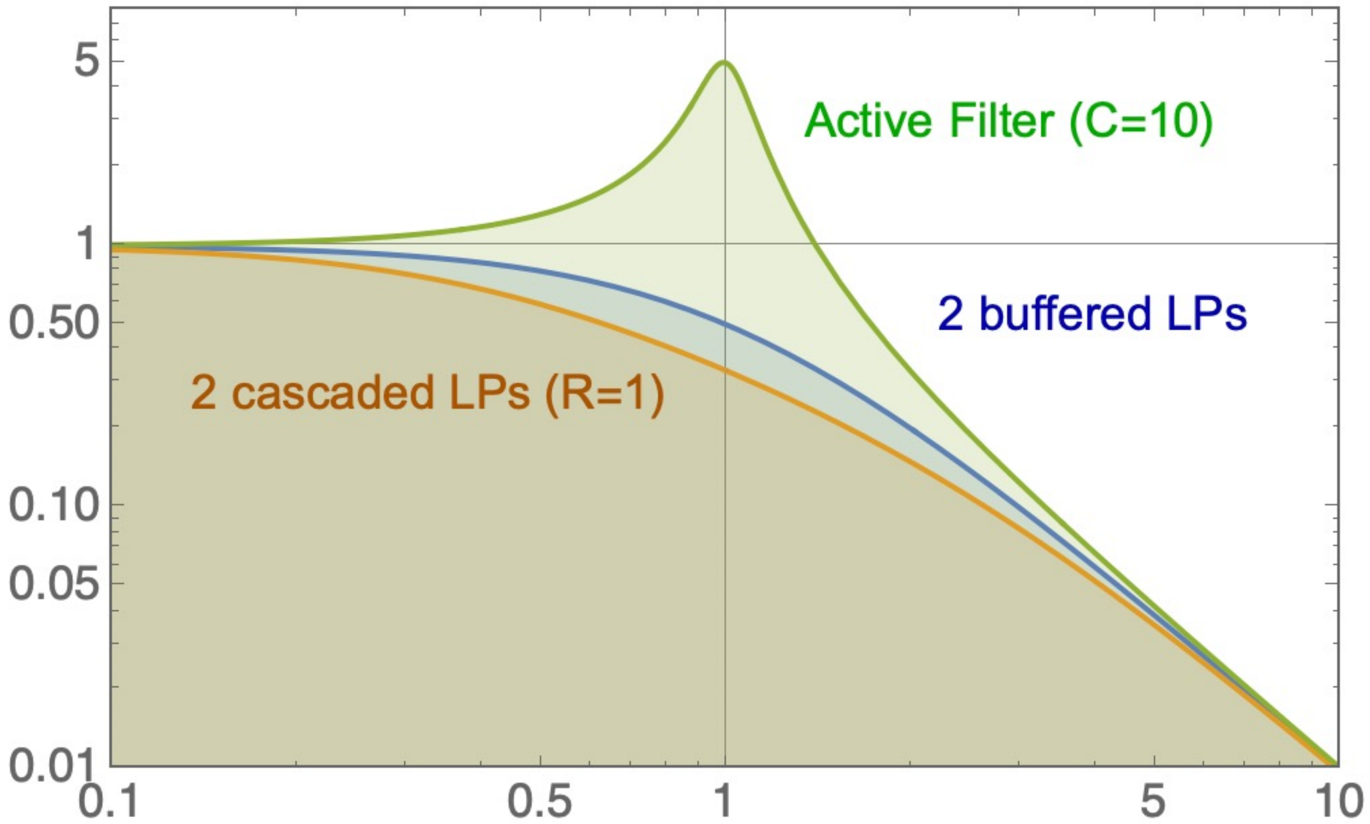
- This transfer function has two ***COMPLEX*** (conjugate) poles:



# Bode Plots of 2nd Order Filters

- The active filter has an overshoot (for the values chosen)
- This is typical for complex conjugate poles



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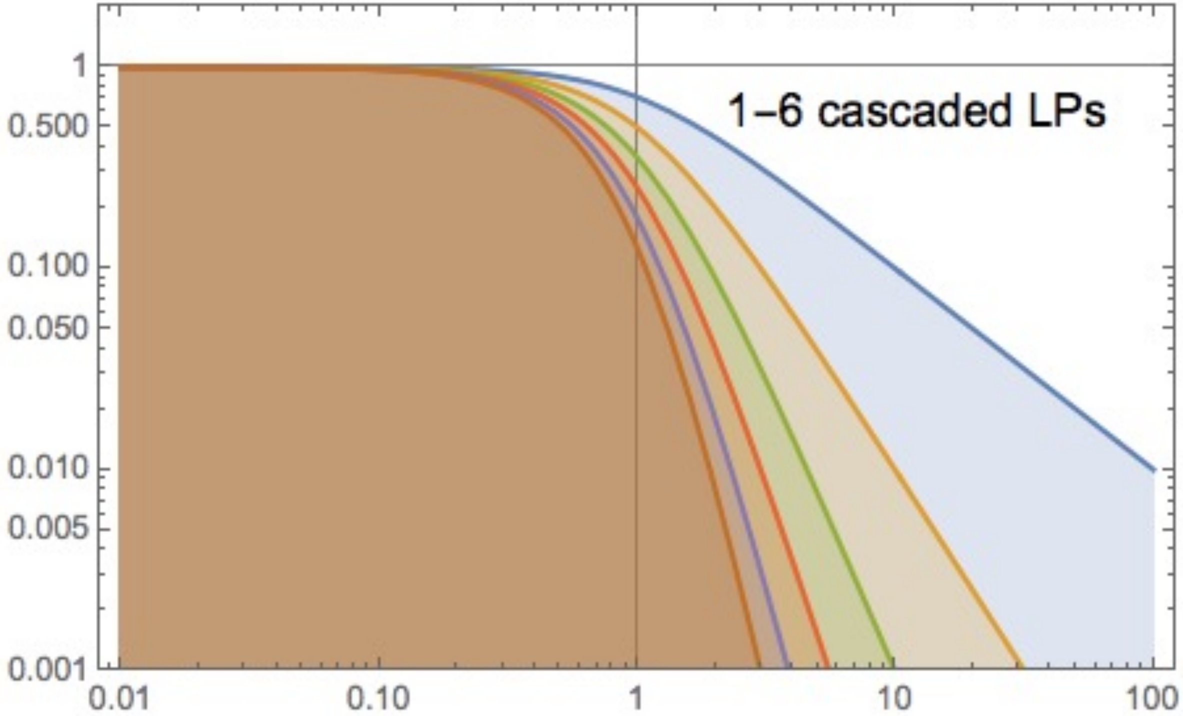
# Making Steep Filters

# A Steep Low Pass Filter

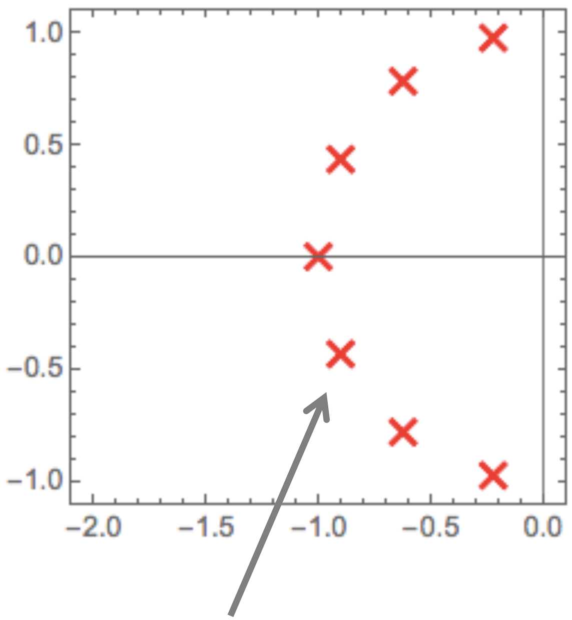
- We want to design a higher ($N^{th}$) order low-pass filter which drops suddenly from **pass band** to **stop band**.
- We know that we roll off with slope –N at the end (for s→∞).



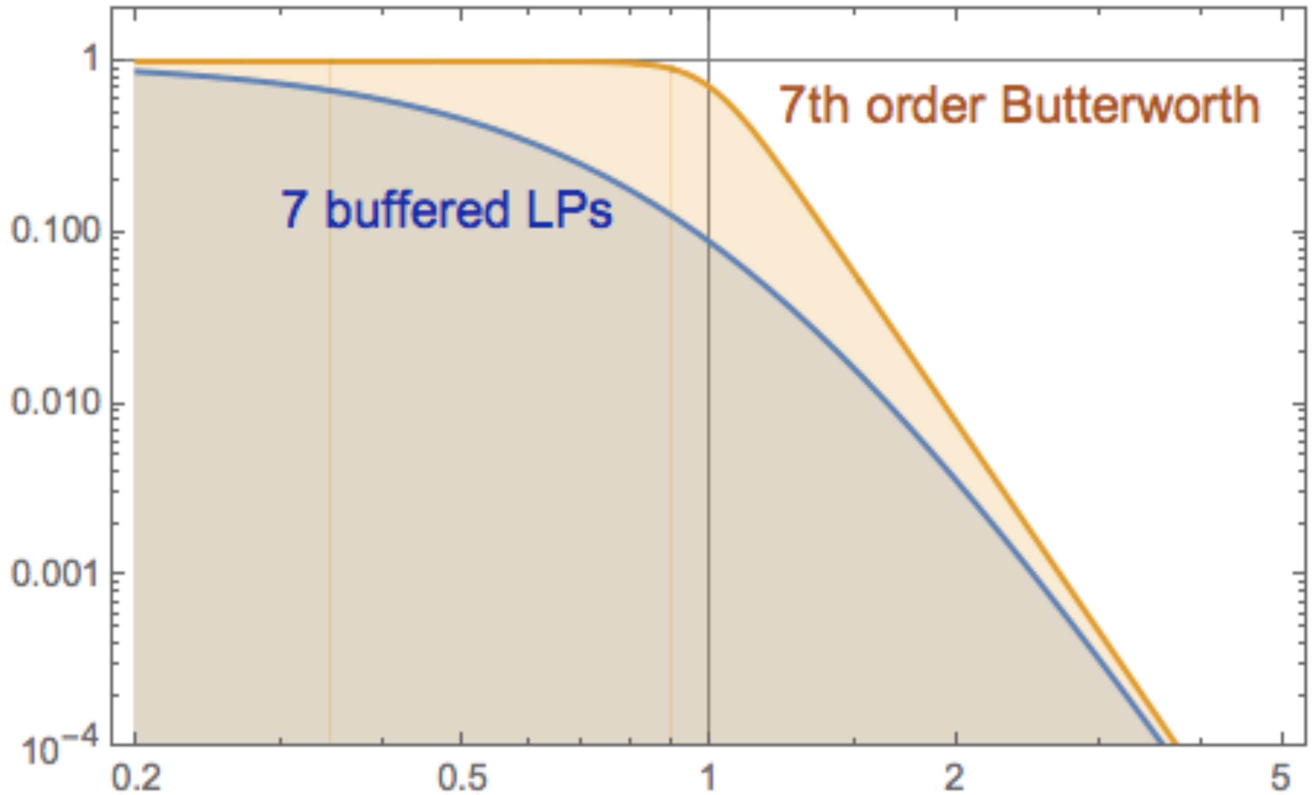

- Simple cascaded LPs attenuate by $2^{-N/2}$ at the corner
- Can this be improved ?

# Choosing the Poles

- The Idea: Use complex poles and adjust them 'somehow'
- 'Butterworth' arranges poles on circle. Here: 7th order.



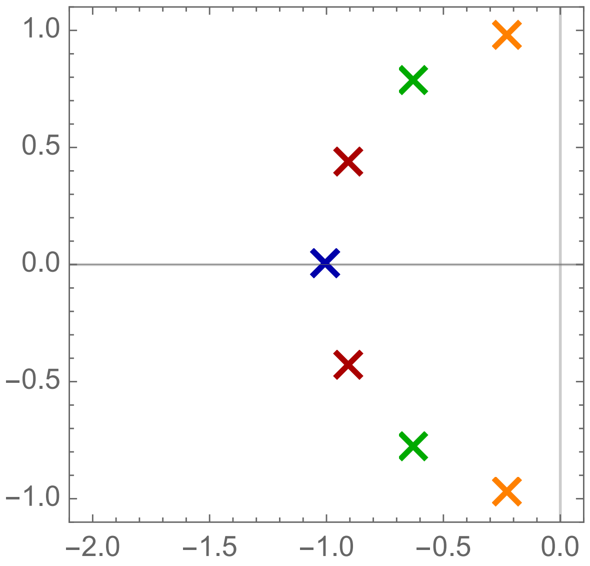

It can be shown (easily) that poles on a circle have same corner frequencies.

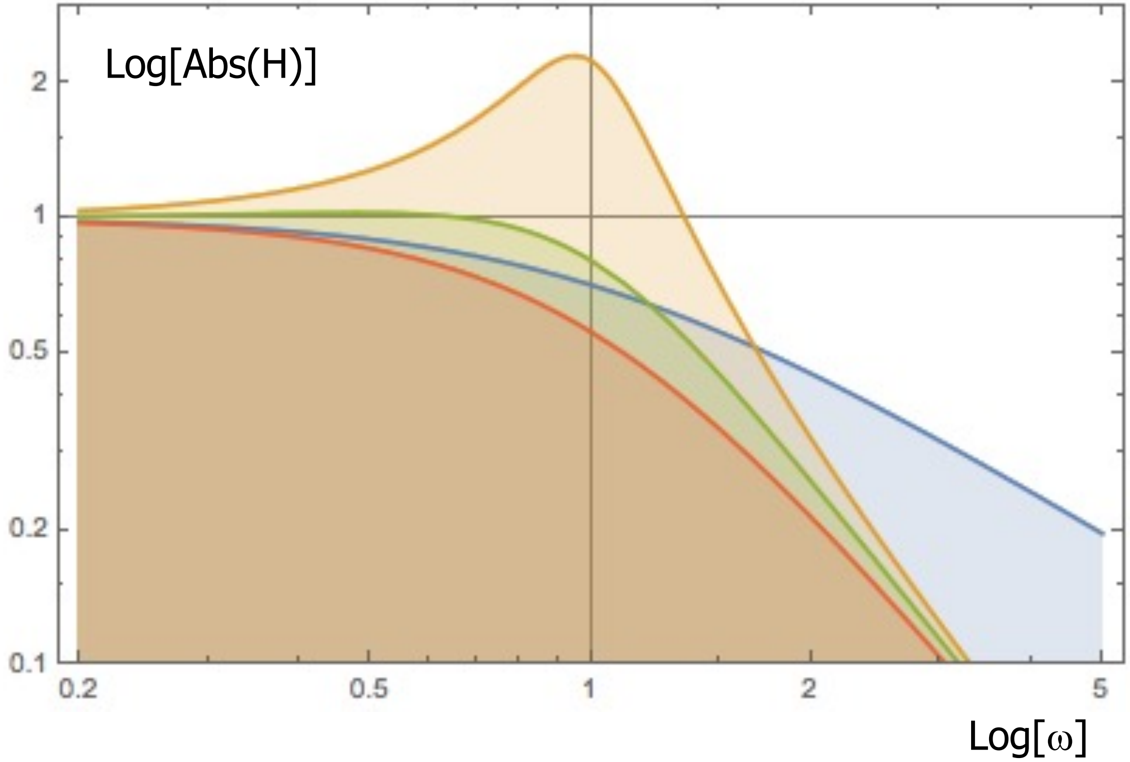



- Wow! Butterworth attenuation at the corner is only -3dB !

# (Decomposing the Butterworth Filter)

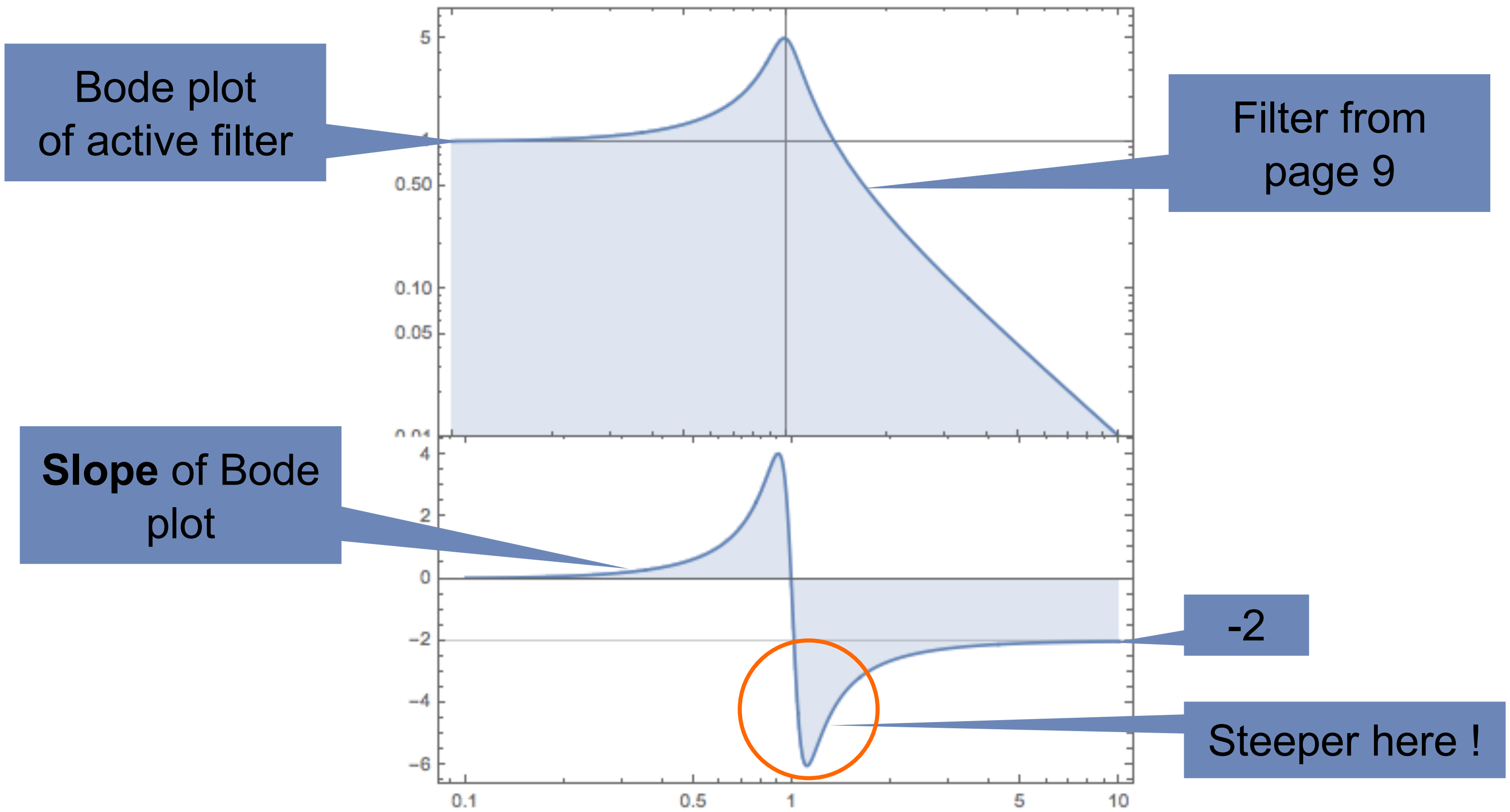
- For N=7:
  - One real pole (1st order, blue)
  - 3 conjugate poles (2nd order)



# Even Steeper?

- Remember: For *large* frequencies, we will *always* roll off with $s^{-N}$ (the order of the filter, i.e. the number of caps)
- But: The 'peaking' for complex poles provides steeper response *close to the bandwidth*:



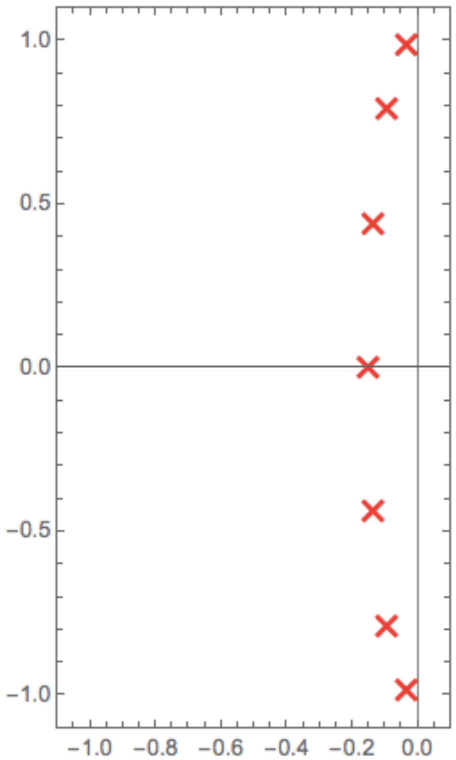
# Placing the Poles...

- There are obviously MANY possibilities to place the poles…

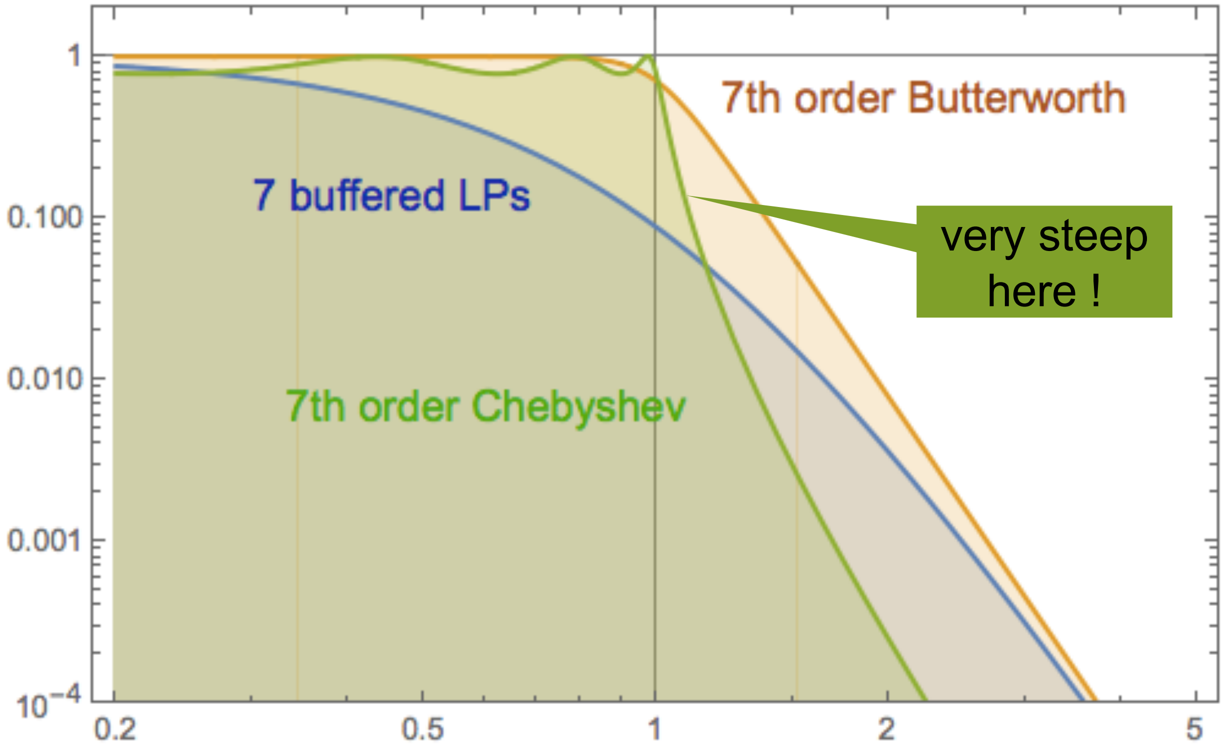
- Desired filter properties are for instance
  - Flatness/ripple of the response in the pass band
  - Steepness of the drop
  - Ripple in the stop band
  - Response to step signals (overshoots)
  - Phase behavior

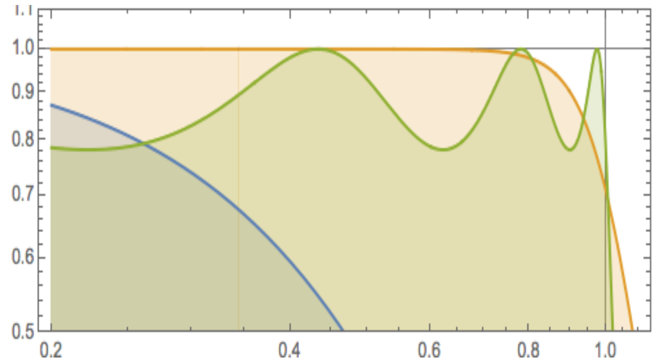
- Four main types have evolved:
  - Butterworth: Flat pass band
  - Bessel: No phase shift, no overshoot
  - Chebyshev: Steeper roll off, but ripple in pass band
  - Elliptic: Even steeper roll off, but ripple in pass and stop band

# Example: A Chebyshev Filter (7th order)




Pole location for a 7th order Chebyshev filter (there are others, depending on the desired pass band ripple)




Zoom of pass band showing ripple of Chebyshev and flat response of Butterworth