Exercise:
Time Domain

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Exercise 1

- A filter is made up of a
  - low-pass
  - an ideal unity gain buffer
  - a high-pass with same corner frequency as the low-pass

- What is the time response of this filter to a unit step ?

- Calculate and Simulate!

Solution 1

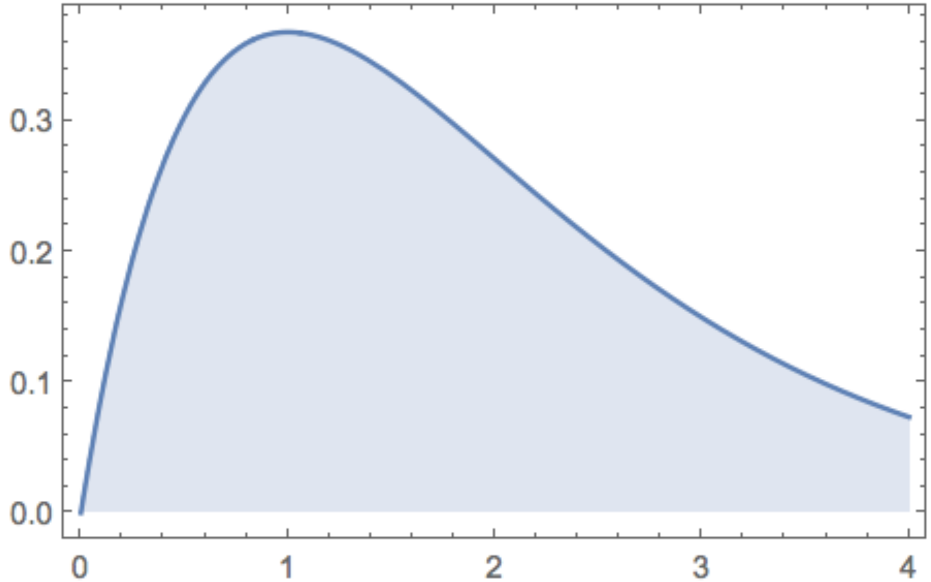
In[14]:= `f[t_] = InverseLaplaceTransform[1/s 1/(1 + s/τ) (s/τ)/(1 + s/τ), s, t]`

$$\mathrm{f}[t\_] = \mathrm{InverseLaplaceTransform}\left[\frac{1}{s}\,\frac{1}{1+\frac{s}{\tau}}\,\frac{\frac{s}{\tau}}{1+\frac{s}{\tau}},\ s,\ t\right]$$

Out[14]= $e^{-t\,\tau}\,t\,\tau$

In[17]:= `Plot[f[t] /. τ → 1, {t, 0, 4}]`

Out[17]=

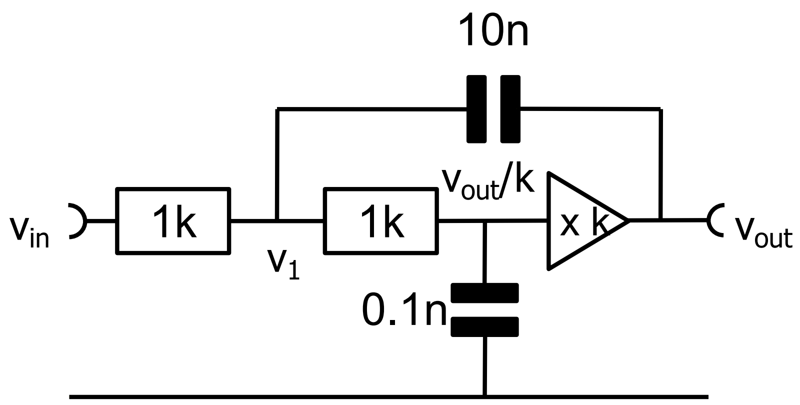


In[19]:= `Solve[f'[t] == 0, t] // First`

Out[19]= $\left\{t \to \frac{1}{\tau}\right\}$

Exercise 2

- Derive H[s] for the following active filter (use Mathematica!)
  - The (ideal) voltage amplifier has gain k



- For which k do you get a divergence ?
- What is the step response for k=1 ?

- Simulate the circuit and compare quantitatively.

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Solution 2

- H[s]

Absolute frequencies do not matter. I use R=1 and C=10

In[221]:= $EQ1 = \frac{vin - v1}{1} == \frac{v1 - \frac{vout}{k}}{1} + (v1 - vout)\, s\, 10;$

$EQ2 = \frac{v1 - \frac{vout}{k}}{1} == \frac{vout}{k}\, s\, \frac{1}{10};$

In[224]:= `$Assumptions = k > 0 && s > 0 && ω > 0;`

In[225]:= `Eliminate[{EQ1, EQ2}, v1] // Simplify`

Out[225]= $(5 + 51\, s + 5\, s^2)\, vout == 5\, k\, (vin + 10\, s\, vout)$

In[226]:= `Solve[%, vout] // First`

Out[226]= $\left\{vout \to -\frac{5\, k\, vin}{-5 - 51\, s + 50\, k\, s - 5\, s^2}\right\}$

In[227]:= $H_{active}[s\_, k\_] = \frac{vout}{vin}$ `/. % // Simplify`

Out[227]= $\frac{5\, k}{5 + (51 - 50\, k)\, s + 5\, s^2}$

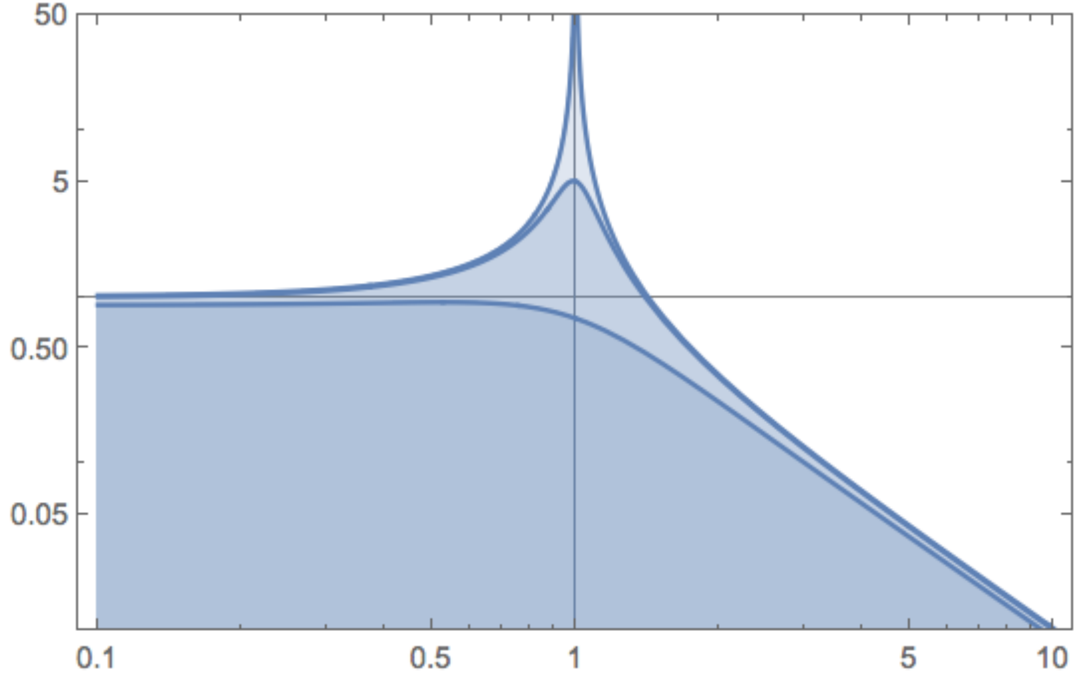
In[228]:= $H_{active}[s, 1]$

Out[228]= $\frac{5}{5 + s + 5\, s^2}$

Solution 2

- The filter produces a high output amplitude peaking at ω=1 (for R,C, chosen).
- The peaking depends on k. Plot 3 values of k:

```
LogLogPlot[{Abs@Hactive[ⅈ ω, {0.9, 1, 1.02}]}, {ω, 0.1, 10},
 Frame → True, PlotRange → {Full, {0.01, 50}}, Filling → Axis,
 GridLinesStyle → Directive[Gray, Thin], GridLines → {{1}, {1}}]
```



Solution 2

- Divergence:
  - The value at the peak is H[i ω] for ω=1:

```
In[249]:= gpeak = Abs@H_active[i, k]
```

Out[249]= $5\,\mathrm{Abs}\left[\frac{k}{51-50\,k}\right]$

```
In[253]:= Solve[51 - 50 k == 0, k] // N

Out[253]= {{k → 1.02}}
```

Solution 2

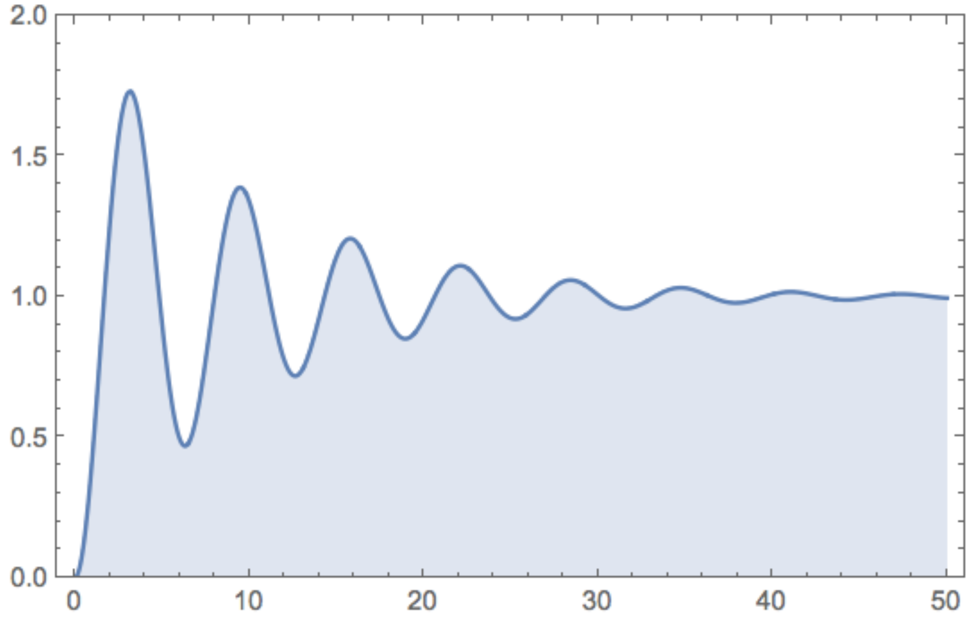
- Step Response

In[322]:=
```
Step1 = InverseLaplaceTransform[Hactive[s, 1] / s, s, t] // Simplify
```

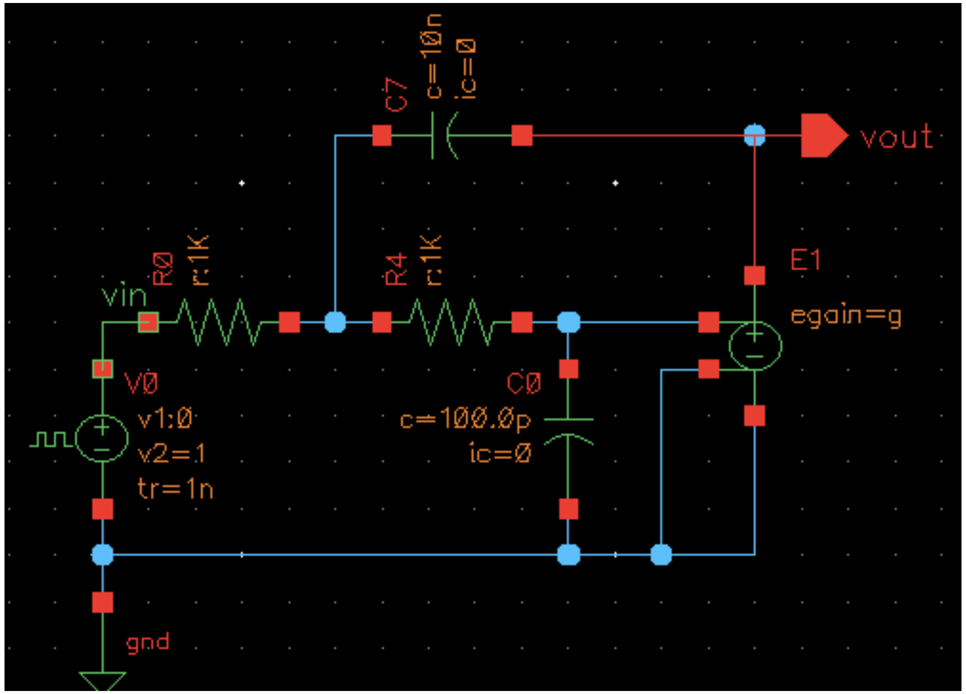
Out[322]= $1 - e^{-t/10} \mathrm{Cos}\left[\frac{3\sqrt{11}\,t}{10}\right] - \frac{e^{-t/10} \mathrm{Sin}\left[\frac{3\sqrt{11}\,t}{10}\right]}{3\sqrt{11}}$

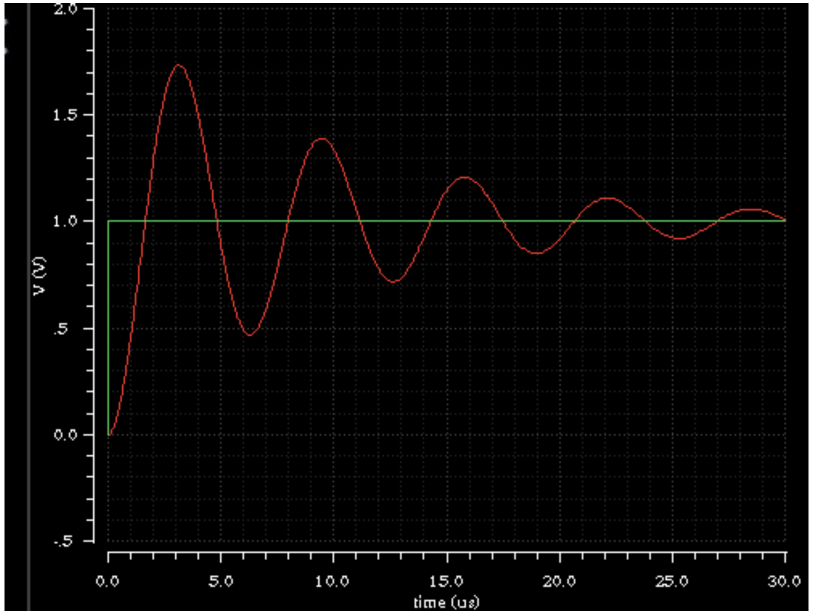
In[287]:=
```
Plot[Step1, {t, 0, 50}, PlotRange → {0, 2}, Frame → True, Filling → Axis]
```

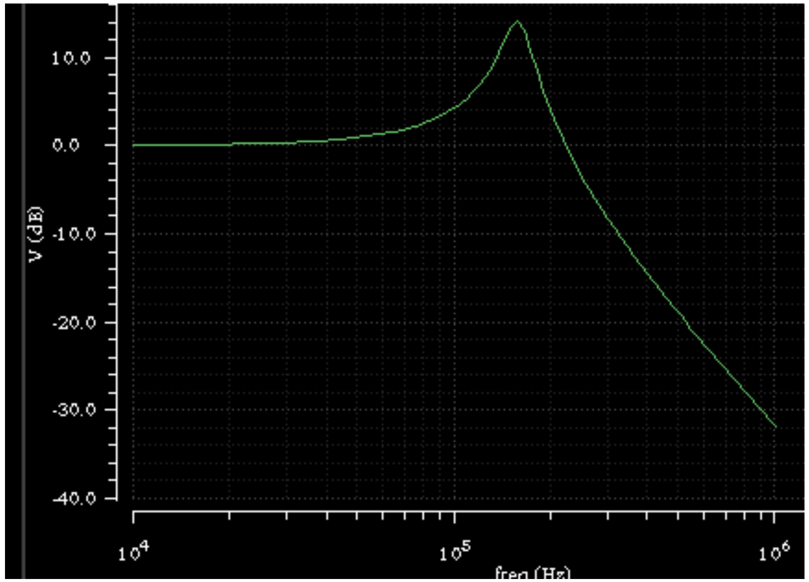
Out[287]=



Simulation:


C7
c=10n
ic=0
vout
vin
R0
r:1K
R4
r:1K
E1
egain=g
V0
v1:0
v2=1
tr=1n
C0
c=100.0p
ic=0
gnd


2.0
1.5
1.0
.5
0.0
-.5
V (V)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
time (us)


10.0
0.0
-10.0
-20.0
-30.0
-40.0
V (dB)
10^4
10^5
10^6
freq (Hz)